COMPUTATION OF THE COMPLIANCE MATRIX FOR ROTARY LIP SEAL

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Abstract

The understanding and the modelling of the ElastoHydroDynamic(EHD)behaviour of lip seals needed more than fifty years of continuous investigations. Even if many studies have been dedicated to the EHD modelling of rotary lip seal,an important aspect has not been well described:any scientific section of the international literature insists on the calculation method of the seal elasticity.

Thus, the scope of this work is to prospects two different methodsused to calculate the influence coefficient matrix: by using Boussinesq approach and by using a specific finite element application developed in Pprime Institute of Poitiers. The resultsshowthat the two approaches leadto important differences concerning the predictionofseal power lost and leakage.

Introduction

The rotary lip seal is the most common type of rotary shaft seals. It’s used to withstand differences in pressure, contain lubricant and exclude contaminants such aswater and dust particles. During the last decades, great efforts have been done to understand and model the lip seal behaviour in application involving rotary mechanisms.

In most studies found in literature,an important aspect of the models used to predict the behaviour of lip seal has not been well described: the calculation method of the “influence coefficient matrix”.

Therefore, the aim of this work is to compare twonumerical calculation methods of the “influence coefficient matrix”. Namely, the Boussinesq method anda Finite Elements (FE) application developed in PprimeInstitute [1]. To perform this study, the following steps are proposed:

- Defining mechanical behaviour of the lip based on elastomer characterization; Hyperelastic (nonlinear) or Elastic (linear behaviour) ,
- Describing the structural analysis of lip and the different way to compute matrix compliance,
- Comparingthe numerical predictions in terms of leakage and power loss, obtained by using the two compliance matrix.

Model

Assumptions

Fig.1 shows a schematic diagram of a typical lip seal and the region near the sealing zone. It is assumed that:

- The seal operates in steady state conditions,
- The viscosity of the lubricant is constant,
- The air side of the seal is flooded with lubricant, so the reverse pumping rate can always be calculated,
- The average film thickness is uniform in the axial direction.

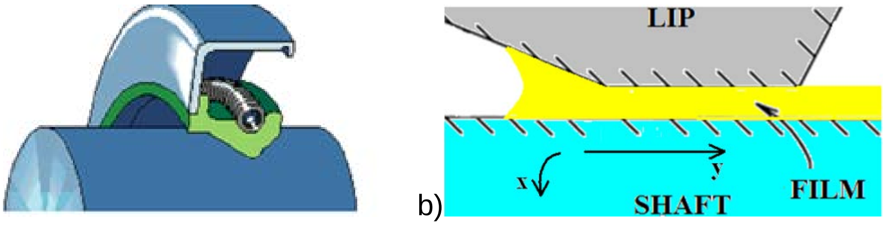
a)  b)

Figure 1: a) Schematic section of radial lip sealb) Schematic diagram of the sealing zone

Governing Equations

In Fig. 1b)*x* is the circumferential direction and *y* is the axial direction. The upper stationary surface represents the lip surface, while the lower moving surface represents the shaft surface.

The Reynolds equation in a Cartesian co-ordinate system takes the form:

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(h^3\frac{\partial p}{\partial y}\right)=6\mu U\frac{\partial h}{\partial x}+12\mu\frac{\partial h}{\partial t} \quad (1)$$

As the shaft or Lip is considered smooth, we assume:

$$\frac{\partial h}{\partial t}=0 \quad (2)$$

In order to take into account cavitation effect, another formulation of equation (1) is developed to deal with the film rupture/replenishment conditions [2]:

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial D}{\partial x}\right)+\frac{\partial}{\partial y}\left(h^3\frac{\partial D}{\partial y}\right)=6\mu U.\frac{\partial h}{\partial x}++6\mu(1-F)\left(U.\frac{\partial D}{\partial x}\right) \quad (3)$$

Where:

$$\begin{cases} D = p \, and F = 1, when p > 0 \\ D = r - h, r = \frac{\rho}{\rho_0} h \, and F = 0, when p \leq 0 \end{cases} \quad (4)$$

ρ and ρ_0 are densities of lubricant-gas mixture and lubricant respectively.

The boundary conditions are:

- $p(x,0)=p_f$ (sealed fluid pressure)
- $p(x,b)=p_0$ (Air pressure)
- p(0,y)=p(λ,y) (Axisymmetric condition)

Film Thickness

$$\{h\} = \{h_2\} + \{h_d\} + h_0 \quad (5)$$

Where :

- *$h_2(x,y)$*:Lip surfacefluctuations (Figure 3),
- *h_0*: Average film thickness,
- *h_d* : Lip deformation, such as$h_d = [C_1].\{p - p_s\}$,where $[C_1]$ compliance matrix and p_s the contact static pressure,

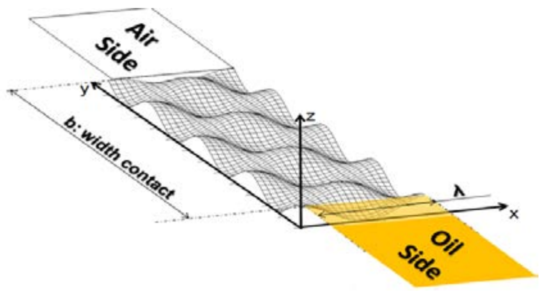


Figure 2 : Deterministic Lip seal and Shaft surface: sinusoidal form

The deterministic form of the lip is given by the following expressions:

- Lip Roughness (Figure 2):

$$h_2(x,y) = \frac{H_2}{2} \cos\left(\frac{2\pi.NBX2}{\lambda}(x - \delta)\right).\left(1 - \cos\left(\frac{2\pi.NBY2}{b} y\right)\right) \quad (6)$$

Where H_2 is the amplitude of the lip surface, NBX2 and NBY2 arethe number of peaks according respectively to circumferential direction x and to axial direction y, $\delta = [C_2].\{\tau\}$ is the tangential lip deformation due to tangential shear stress.

Structural mechanic analysis

Knowing that a numerical analysis starts with the evaluation of the static dry pressure profile and contact length, figure 3 shows the FE model of the seal. The model is meshed with axisymmetric stress elements. The computations are made in large displacement and deformation hypotheses. The shaft is usually made in more rigid material (typically steel) than the elastomeric seal. Consequently, it is reasonable to consider the shaft as an analytical defined rigid element.

This result is the starting point of the EHD modelling: the integration of the contact pressure gives the force that must be balanced by the hydrodynamic pressure. The axial contact length defines the study domain length in the axial direction. The second length is chosen equal to the roughness periodicity in the circumferential direction.

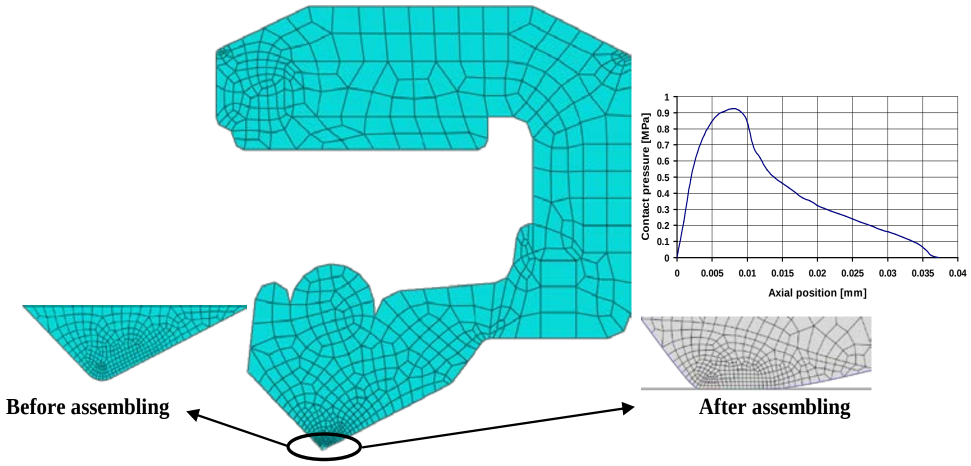


Figure.3: FE model of the seal

Hyperelastic / Elastic equivalence

Rare are the studies that have treated of the equivalence between the elastic and hyperelastic model for a rotary lip seal.In the following, the young modulus E and the Poisson ratio vare approximated from the Mooney-Rivlin parameters, such as:

$$\begin{cases} E = 4(1+\nu)(C_{10}+C_{01}) \\ K = \dfrac{2}{D} = \dfrac{E}{3(1-2\nu)} \end{cases} \quad (7)$$

Fig 4 shows the differences in terms of contact width and maximum contact pressure, between simulations performed by considering a hyperelastic behavior and the equivalent Hookean model. The differences are estimated for different interferences ranging from4µmup to 450 µm. The hyperelastic model parameters are: C_{10} = -2.746MPa, C_{01} = 4.597MPa and D = 0.001MPa^{-1}.The equivalent Young modulus is E = 11.1 MPa with **v** = 0.499. It can be observed that the increase of the interference leads to the increase of the differences between the two cases. This proves that a precise evaluation of the elastomer behaviour is necessary, especially when the lip/shaft interference is important.

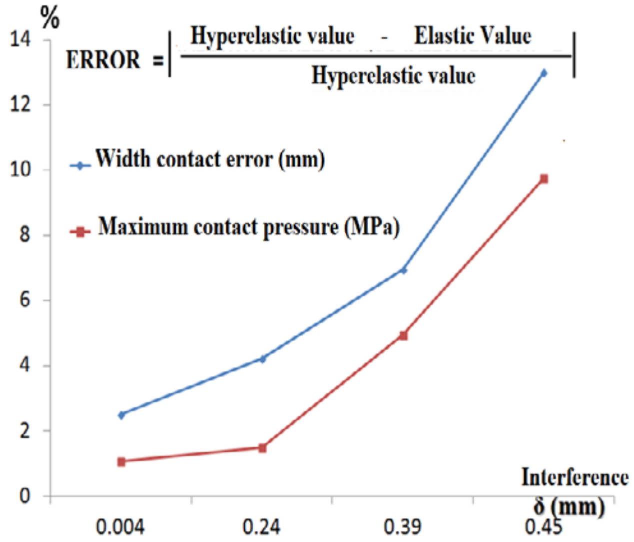


Figure 4 :Curve equivalence between Elastic and Hyperelastic model

Compliance matrix: FE method

Before computing the compliance matrix, the following hypothesis is made: the radial strain imposed by the fluid film in contact is small in comparison with the radial strain imposed by the seal/shaft interference. Therefore, the elastic response of the seal is computed as a linear perturbation of the mounted seal: the seal material is a classical Hookean model and the computations are made in small displacement and deformation hypotheses.

Moreover, the lip is considered to have, along a height *d*,a 3D behaviour. The elastic deformation of the lip is treated by FE method using elements with twenty nodes for the 3D part and eight nodes 2D elements for the rest of the seal structure (see Figure 5). In order to take into account the global axisymmetric hypothesis, rigid beams connect the two faces of the 3D domain, giving the same displacement for the connected nodes. Two compliance matrixes [C1] and [C2] are calculated. [C1] is used to compute the radial displacement and [C2] is used to compute the circumferentially tangential displacement.

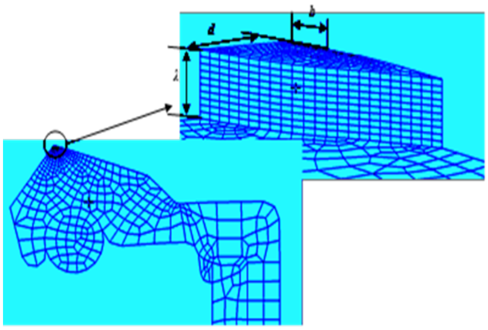


Figure 5: Lip seal 3D finite element model

Compliance matrix:Analytical Method

This is a simplest method based on the Boussinesq-Love approach[3]. The method supposes that the lip seal elastic behaviour can be approachedby an elastic half-space domain. Therefore, the deformation field can be estimated by (see figure.6):

$$d(x,y) = \frac{1-\upsilon^2}{E}\iint \frac{p(x_j,y_j)dx_j.dy_j}{\sqrt{(x-x_j)^2+(y-y_j)^2}} \tag{8}$$

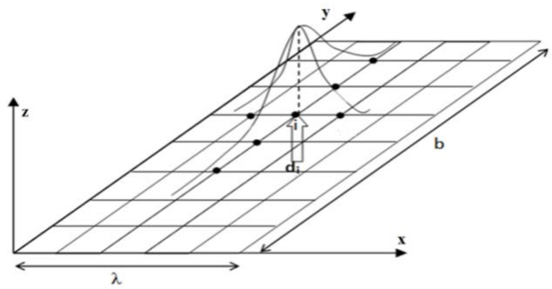


Figure 6: Boussinesq model

The relationship between, the fields of displacement [d], and pressure [p], is given by:

$$[d] = [C].[p] \tag{9}$$

$$[C] = \begin{bmatrix} C_{11} & \cdots & C_{1j} & \cdots & C_{1N_N} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ C_{i1} & \cdots & C_{ij} & \cdots & C_{iN_N} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ C_{N_N i} & \cdots & C_{N_N j} & \cdots & C_{N_N N_N} \end{bmatrix} \tag{10}$$

With$C_{i,j} = \frac{1-\upsilon^2}{\pi E}$

$$\Big\{(y_i - y_j - d).\ln\left(\frac{x_i - x_j - c + \sqrt{(y_i - y_j - d)^2 + (x_i - x_j - c)^2}}{x_i - x_j + c + \sqrt{(y_i - y_j - d)^2 + (x_i - x_j + c)^2}}\right)$$

$$+ (y_i - y_j + d).\ln\left(\frac{x_i - x_j + c + \sqrt{(y_i - y_j + d)^2 + (x_i - x_j + c)^2}}{x_i - x_j - c + \sqrt{(y_i - y_j + d)^2 + (x_i - x_j - c)^2}}\right)$$

$$+(x_i - x_j + c).\ln\left(\frac{y_i - y_j + d + \sqrt{(y_i - y_j + d)^2 + (x_i - x_j + c)^2}}{y_i - y_j - d + \sqrt{(y_i - y_j - d)^2 + (x_i - x_j + c)^2}}\right)$$

$$+ (x_i - x_j - c).\ln\left(\frac{y_i - y_j - d + \sqrt{(y_i - y_j - d)^2 + (x_i - x_j - c)^2}}{y_i - y_j + d + \sqrt{(y_i - y_j + d)^2 + (x_i - x_j - c)^2}}\right)\Big\} \quad (11)$$

whered=b/2M , c=λ/2N,*N* is thenode number according to *x* direction, M is thenode number according to*y* direction and NN isthe total number of nodes.Due to the global axisymmetric, the deformations are computed in the middle nodes domain and deduced by translation to the rest of the domain (figure 7).

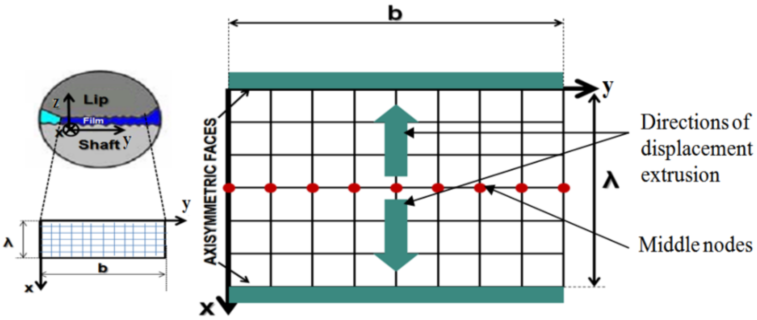


Figure 7: Boussinesq model corrected by extrusion displacement axisymmetric faces

Validation

As indicated in a previous paragraph, in order to compute the compliance matrix, it is assumed that the elastic response of the seal is computed as a linear perturbation of the mounted seal. So, to validate this hypothesis, the radial deformation under a uniform unitary pressure field is computed for both used methods and then compared with a FE results obtained without any simplification (non-linear model). As the model is axisymmetric, the obtained deformation is constant through the circumferential direction and only the axial variation is represented in figure8. It can be observed that the linear FE model and the non-linear FE model give almost the same elastic response. However, the Boussinesq-Love method leads to very different results.

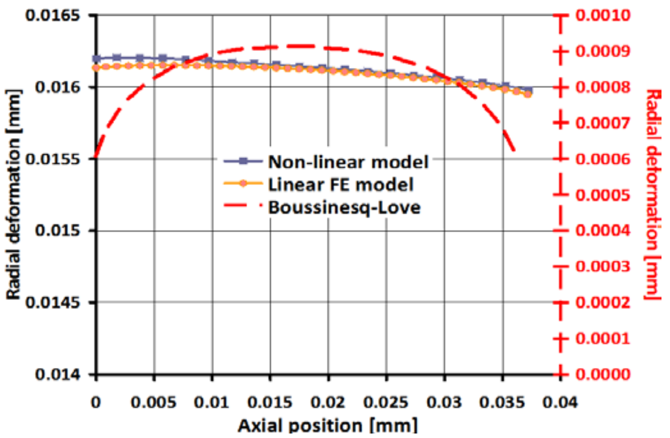


Figure 8: Radial deformation of the lip seal surface under a constant pressure (1 MPa)

The comparison between the two presented approaches used to compute the compliance matrix is extended to the EHD computation of the seal. Figure 9 shows the ratio between the seal functioning parameters computed analytically by the Boussinesq method and numerically by FE. It can be noted that, even if easier to implement, the analytical method leads to a under estimation of the leakage (up to 50% in the presented case) and to an over estimation of the power loss (up to 20%).

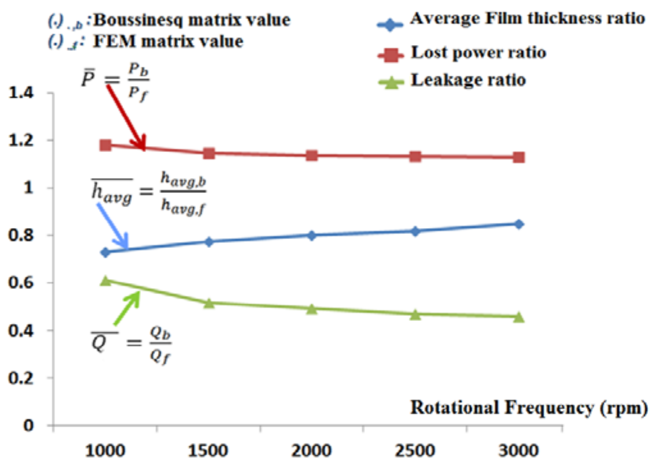


Figure 9 : Influence of the compliance matrix computation method over the seal functioning parameters

Conclusion

The structural analysis is required to initiate the EHDmodellingof a lip seal. We have shown in this study that the equivalence between the linear and the hyperelastic model can be verified only for small displacement (when the interference between the shaft and the seal is small).

After determination of the contact width, we explained two different methods of calculating the compliance matrix. Then, we have shown that the analytical approach gives deformations10 times smaller than calculated by Abaqus, without any simplification. In the same time, the numerical method based on FE computation leads to accurate results. It is next proved that these large differences in stiffness between the two models lead to important differences in terms of leakage and power loss predictions, which proves that only the numerical method must be used in EHD modelling of rotary lip seals.

Acknowledgments

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References

[1]. R.F.Salant, “Modelling rotary lip seals”, Wear 207, (1997) 92-99.
[2]. M.Hajjam,D.Bonneau,”A transient Finite element cavitation algorithme with application to radial lip seals”, Tribology international (2007),1258-1269.
[3]. S.K.Tonder, R.F.Salant,”Non-leaking lip seals: A roughness effect”, Journal of tribology (1992),114-595.